
Two-stage column generation and applications in container terminal management

Ilaria Vacca

Matteo Salani

Michel Bierlaire

Transport and Mobility Laboratory

EPFL

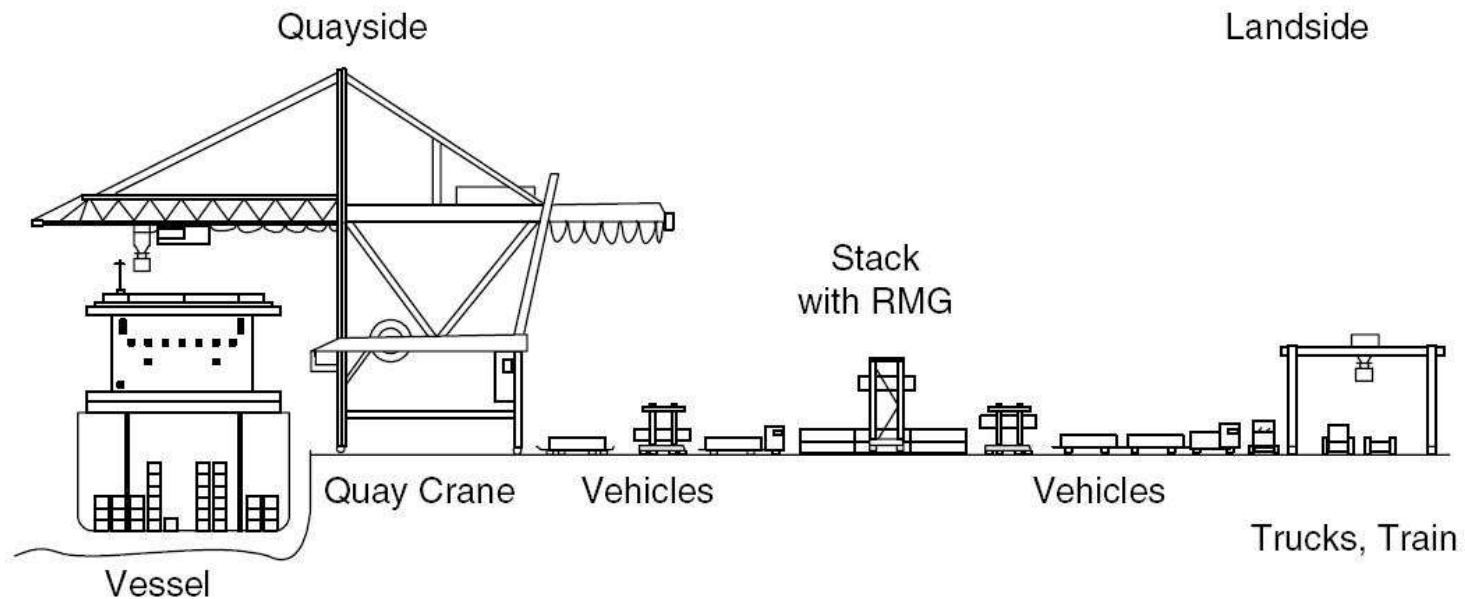
8th Swiss Transport Research Conference (STRC)

October 16, 2008

Outline

- Motivation : Optimization of container terminal operations
- Tactical Berth Allocation Problem (TBAP) with Quay Crane Assignment
- Methodology : Two-stage column generation
- General framework for large-scale problems
- Illustration on TBAP
- Conclusions & future work

Container terminals



Source: Steenken et al. (2004)

Tactical Berth Allocation with QCs Assignment

Giallombardo, Moccia, Salani and Vacca (2008)

Problem description

- *Tactical Berth Allocation Problem* (TBAP): assignment and scheduling of ships to berths, according to time windows for both berths and ships;
- *Quay-Cranes Assignment Problem* (QCAP): a quay crane (QC) profile (number of cranes per shift, ex. 332) is assigned to each ship.

Issues

- the chosen profile determines the ship's handling time and thus impacts on the scheduling;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).

Tactical Berth Allocation with QCs Assignment

Find

- a berth allocation
- a schedule
- a quay crane assignment

Given

- time windows on availability of berths
- time windows on arrival of ships
- *handling times dependent on QC profiles*
- values of QC profiles

Aiming to

- maximize total value of QC assignment
- minimize housekeeping costs of transshipment flows between ships

Tactical Berth Allocation with QCs Assignment

Remark

- the MILP formulation proposed by Giallombardo et al. (2008) is hardly solvable already on small instances;

Column generation approach

- Dantzig-Wolfe reformulation and master problem;
- Resource Constrained Elementary Shortest Path pricing subproblem;

Issues

- the pricing subproblem is unmanageable due to the huge size of the underlying network;
- the complexity is given by the combinatorial number of decision variables in the original formulation (profile assignment variables).

Two-stage column generation

Salani and Vacca (2008)

Context

Dantzig-Wolfe (DW) reformulation of combinatorial problems.

Motivation

Many problems exhibit a *compact* formulation with many variables (possibly an exponential number) which results in an unmanageable associated pricing problem, when the extensive formulation is obtained through DW.

Similar problems, in addition to TBAP:

- Vehicle Routing Problem (VRP) with Discrete Split Delivery
- Field Technician Scheduling Problem
- Routing helicopters for crew exchanges on off-shore locations

DW decomposition in Integer Programming (IP)

Original or Compact Formulation (CF)

$$z_{IP} = \min \quad c^T x \quad (1)$$

$$s.t. \quad Ax \geq b, \quad (2)$$

$$Dx \geq d, \quad (3)$$

$$x \in \mathbb{Z}_+^n. \quad (4)$$

Assumptions:

- linear relaxation of CF provides a weak lower bound;
- constraints $\{Dx \geq d\}$ can be easily convexified.

DW decomposition in Integer Programming (IP)

Let $P = \text{conv}\{x \in \mathbb{Z}_+^n : Dx \geq d\} \neq \emptyset$ be a bounded polyhedron. Each $x \in P$ can be represented as a convex combination of extreme points $\{p_q\}_{q \in Q}$ of P .

Extensive Formulation (EF)

$$z_{IP} = \min \sum_{q \in Q} c_q \lambda_q \quad (5)$$

$$s.t. \quad \sum_{q \in Q} A_q \lambda_q \geq b, \quad (6)$$

$$\sum_{q \in Q} \lambda_q = 1, \quad (7)$$

$$\lambda \geq 0, \quad (8)$$

$$x = \sum_{q \in Q} p_q \lambda_q, \quad (9)$$

$$x \in \mathbb{Z}_+^n. \quad (10)$$

where $c_q = c^T p_q$ and $A_q = Ap_q \quad \forall q \in Q$.

Standard column generation

The integrality of x in (EF) is relaxed.

Master Problem (MP)

$$z_{MP} = \min \sum_{q \in Q} c_q \lambda_q \quad (11)$$

$$s.t. \quad \sum_{q \in Q} A_q \lambda_q \geq b, \quad (12)$$

$$\sum_{q \in Q} \lambda_q = 1, \quad (13)$$

$$\lambda \geq 0. \quad (14)$$

Standard column generation

Column generation (CG)

The so-called *Restricted Master Problem* (RMP) is repeatedly solved on a subset of variables λ , which otherwise would be an exponential number.

At each iteration of column generation we add profitable variables not yet in the formulation by solving the *pricing subproblem*:

$$\tilde{c}_q^* = \min_{q \in Q} \{ \tilde{c}_q := c_q - \pi A_q - \pi_0 \} \quad (15)$$

- if $\tilde{c}_q^* \geq 0$, there is no negative reduced cost column and the algorithm terminates;
- if $\tilde{c}_q^* < 0$ and finite, we add the column to the RMP and iterate.

References: Lübbecke and Desrosiers (2005), Desaulniers et al. (2005).

Two-stage column generation

Context

Compact formulation with a huge number of variables; standard column generation not efficient because the resulting pricing subproblem is unmanageable.

Novel idea

Develop a framework in which a combinatorial problem is solved starting from a *Partial Compact Formulation* (PCF), with the same approach used in column generation for the restricted master problem, obtaining a *Partial Master Problem* (PMP).

Two-stage column generation

Partial Compact Formulation (PCF)

$$\bar{z}_{IP} = \min \quad \bar{c}^T \bar{x} \quad (16)$$

$$s.t. \quad \bar{A}\bar{x} \geq b, \quad (17)$$

$$\bar{D}\bar{x} \geq d, \quad (18)$$

$$\bar{x} \in \mathbb{Z}_+^{\bar{n}}. \quad (19)$$

Remarks:

- X is the set of compact formulation variables, $|X| = n$;
- subset $\bar{X} \subset X$, $|\bar{X}| = \bar{n}$ such that (CF) is feasible;
- variables $\hat{X} := X \setminus \bar{X}$ not in the formulation; possibly added via column generation.

Two-stage column generation

Let $\bar{P} = \text{conv}\{\bar{x} \in \mathbb{Z}_+^{\bar{n}} \mid \bar{D}\bar{x} \geq d\} \neq \emptyset$ be a bounded polyhedron. Each $\bar{x} \in \bar{P}$ can be represented as a convex combination of extreme points $\{p_q\}_{q \in \bar{Q}}$ of \bar{P} :

$$\bar{x} = \sum_{q \in \bar{Q}} p_q \lambda_q, \quad \sum_{q \in \bar{Q}} \lambda_q = 1, \quad \lambda \in \mathbb{R}_+^{|\bar{Q}|} \quad (20)$$

Master Problem (MP)

$$\bar{z}_{MP} = \min \quad \sum_{q \in \bar{Q}} \bar{c}_q \lambda_q \quad (21)$$

$$s.t. \quad \sum_{q \in \bar{Q}} \bar{A}_q \lambda_q \geq b, \quad (22)$$

$$\sum_{q \in \bar{Q}} \lambda_q = 1, \quad (23)$$

$$\lambda \geq 0. \quad (24)$$

Two-stage column generation

Algorithm 1: Two-stage column generation

input set \bar{X}

repeat

repeat

 | CG1: generate extensive variables λ for partial master problem (PMP)

until *optimal partial master problem (PMP)* ;

 CG2: generate compact variables x for partial compact formulation (PCF)

until *optimal master problem (MP)* ;

- in (CG1) standard column generation applies; in particular, the dual optimal vector π is known at every iteration and thus reduced costs $\tilde{c}_q := \bar{c}_q - \pi \bar{A}_q - \pi_0$ of λ variables can be directly estimated.
- in (CG2) we need to know the reduced costs of variables $x_i \in \hat{X}$ in order to identify the profitable ones to be added to the partial compact formulation, if any.

Reduced costs of compact variables

- [Walker \(1969\)](#): method which can be applied when the pricing problem is a pure linear program.
- [Poggi de Aragão and Uchoa \(2003\)](#): coupling constraints in the master problem formulation.
- [Irnich, Desaulniers, Desrosiers and Hadjar \(2007\)](#): reduced costs estimation based on paths (not directly applicable to the two-stage framework).
- [Salani and Vacca \(2008\)](#): reduced costs estimation obtained through complementary slackness conditions, applicable to general compact formulations.

Tactical Berth Allocation with QCs Assignment

Giallombardo, Moccia, Salani and Vacca (2008)

MILP Formulation

- $n = |N|$ ships with time windows on the arrival time at the terminal;
- $m = |M|$ berths with time windows on availability;
- a planning time horizon discretized in $|H|$ time steps;
- a set P_i of feasible QC assignment profiles defined for every ship $i \in N$;
- the maximum number Q of quay cranes available in the terminal.

Compact formulation decision variables

- x_{ij}^k : flow variables (scheduling);
- λ_{ip} : profile assignment variables;
- T_i^k : time variables

Tactical Berth Allocation with QCs Assignment

DW Reformulation

- concept of *berth sequence*, which represents a sequentially ordered (sub)set of ships in a berth with an assigned QC profile;
- Ω^k : set of all feasible sequences r for berth $k \in M$;
- z_r^k : decision variable of the extensive formulation which is 1 if sequence $r \in \Omega^k$ is used by berth k and 0 otherwise.

Extensive Formulation

$$\max \sum_{r \in \Omega} v_r z_r \quad (25)$$

$$\text{s.t.} \quad \sum_{r \in \Omega} y_{ir} z_r = 1 \quad \forall i \in N, \quad (26)$$

$$\sum_{r \in \Omega} q_r^h z_r \leq Q^h \quad \forall h \in H, \quad (27)$$

$$\sum_{r \in \Omega} z_r = m, \quad (28)$$

$$z_r \in \{0, 1\} \quad \forall r \in \Omega. \quad (29)$$

Tactical Berth Allocation with QCs Assignment

Reduced cost of a sequence $r \in \Omega$

$$\tilde{v}_r = v_r - \sum_{i \in N} \pi_i y_{ir} - \sum_{h \in H} \mu_h q_r^h - \pi_0 \quad (30)$$

where π_i represents the dual price of serving ship i in sequence r and μ_h represents the dual price of using an additional quay crane at time step h .

Pricing subproblem

$$\max_{r \in \Omega \setminus \Omega'} \{\tilde{v}_r\} = \max_{r \in \Omega \setminus \Omega'} \{v_r - \sum_{i \in N} \pi_i y_{ir} - \sum_{h \in H} \mu_h q_r^h\} - \pi_0 \quad (31)$$

The column r^* with maximum reduced cost is identified. If $\tilde{v}_{r^*} > 0$, we have identified a new column to enter the basis; if $\tilde{v}_{r^*} \leq 0$, we have proven that the current solution of RMP is also optimal for MP.

Tactical Berth Allocation with QCs Assignment

Pricing subproblem

In the pricing problem:

$$\max_{r \in \Omega \setminus \Omega'} \{\tilde{v}_r\} = \max_{r \in \Omega \setminus \Omega'} \left\{ v_r - \sum_{i \in N} \pi_i y_{ir} - \sum_{h \in H} \mu_h q_r^h \right\} - \pi_0 \quad (32)$$

several decisions have to be made:

- (i) whether ship i is in the sequence or not; this decision is represented by cost component y_{ir} ;
- (ii) whether profile p is used by ship i or not; this decision, represented by λ_{ir}^p , is implicitly included in the pricing problem through cost component $v_r = \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_{ir}^p$;
- (iii) the order of ships in the sequence; this decision is implicitly represented by cost component q_r^h .

Tactical Berth Allocation with QCs Assignment

Complexity of the pricing subproblem

- Elementary Shortest Path Problem with Resource Constraints (ESPPRC);
- network with one node for every ship $i \in N$, for every profile $p \in P_i$ and for every time step $h \in H$; arcs have transit time equals to the length of the profile;
- the associated network is huge \rightarrow solving ESPPRC is impractical!

Two-stage column generation

- partial compact formulation defined over a subset $P'_i \subset P_i$ of quay crane profiles for every ship $i \in N$;
- among the quay cranes profiles not yet considered, we select the subset of profiles with strictly positive reduced cost and we iterate the entire process.

Conclusion & future work

Main contribution

- a novel framework to tackle large-scale optimization problems

Advantages

- the pricing problem is easier to solve
- possibly many sub-optimal compact variables are left out from the formulation

Ongoing work

- computational tests

References

- Desaulniers, G., Desrosiers, J. and Solomon, M. (eds) (2005). *Column Generation*, GERAD 25th Anniversary Series, Springer.
- Giallombardo, G., Moccia, L., Salani, M. and Vacca, I. (2008). The tactical berth allocation problem with quay crane assignment and transshipment-related quadratic yard costs, *Proceedings of the European Transport Conference (ETC)*.
- Irnich, S., Desaulniers, G., Desrosiers, J. and Hadjar, A. (2007). Path reduced costs for eliminating arcs, *Technical Report G-2007-79*, Les cahiers du GERAD, HEC Montréal.
- Lübbecke, M. E. and Desrosiers, J. (2005). Selected topics in column generation, *Operations Research* **53**: 1007–1023.
- Poggi de Aragão, M. and Uchoa, E. (2003). Integer program reformulation for robust branch-and-price algorithms, *Proceedings of Mathematical Programming in Rio: A conference in honour of Nelson Maculan*, pp. 56–61.
- Salani, M. and Vacca, I. (2008). Two-stage column generation and applications, *Technical Report TRANSP-OR 081002*, Transport and Mobility Laboratory, EPFL.

Steenken, D., Voss, S. and Stahlbock, R. (2004). Container terminal operation and operations research - a classification and literature review, *OR Spectrum* **26**: 3–49.

Walker, W. E. (1969). A method for obtaining the optimal dual solution to a linear program using the Dantzig-Wolfe decomposition, *Operations Research* **17**: 368–370.